Promoting the Mathematical Success of Emergent Bilinguals Through Problem Solving

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STANDARDS FOR MATHEMATICAL PRACTICE[1,2]

PRACTICE	DESCRIPTION
1. Make sense of problems and persevere in solving them	Students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. Students analyze givens, constraints, relationships, and goals. Students make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. Students monitor and evaluate their progress and change course if necessary.
2. Reason abstractly and quantitatively	Students make sense of quantities and their relationships in problem situations. Students use two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* (abstract thinking and symbolic representation) and the ability to *contextualize* (to pause in order to probe into the referents for the symbols involved).
3. Construct viable arguments and critique the reasoning of others	Students understand and use stated assumptions, definitions, and previously established results in constructing arguments. Students make conjectures and build a logical progression of statements to explore the truth of their conjectures. Students analyze situations by breaking them into cases, and can recognize and use counterexamples. Students justify their conclusions, communicate them to others, and respond to the arguments of others.
4. Model with mathematics	Students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation.
5. Use appropriate tools strategically	Students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Students detect possible errors by strategically using estimation and other mathematical knowledge.
6. Attend to precision	Students try to communicate precisely to others and try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
7. Look for and make use of structure	Students look closely to discern a pattern or structure. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective.
8. Look for and express regularity in repeated reasoning	Students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

[1] Description adapted from: National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards*. Washington, DC: Authors.
[2] Handout prepared for the TODOS Conference 2014. Celedón-Pattichis, S., & Musanti, S. I. *Promoting the Mathematical Success of Emergent*

DOMAIN/STANDARDS FOR GRADE K-3[3]

KINDERGARTEN	FIRST GRADE	SECOND GRADE	THIRD GRADE
Counting and Cardinality • Know number names and the count sequence • Count to tell the number of objects • Compare numbers			
Operations and Algebraic Thinking • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Operations and Algebraic Thinking • Understand and apply properties of operations and the relationship between addition and subtraction • Add and subtract within 20 • Work with addition and subtraction equations • Represent and solve problems involving addition and subtraction	Operations and Algebraic Thinking • Represent and solve problems involving addition and subtraction • Add and subtract within 20 • Work with equal groups of objects to gain foundations for multiplication	Operations and Algebraic Thinking • Represent and solve problems involving multiplication and division • Understand properties of multiplication and the relationship between multiplication and division • Multiply and divide within 100 • Solve problems involving the four
Number and Operations in Base Ten • Work with numbers 11-19 to gain foundations for place value	Number and Operations in Base Ten • Extend the counting sequence • Understand Place value • Use place value understanding and properties of operations to add and subtract	Number and Operations in Base Ten • Understand place value • Use place value understanding and properties of operations to add and subtract	Number and Operations in Base Ten • Use place value understanding and properties of operations to perform multi-digit arithmetic Number and Operations • Fractions (Limited to fractions with denominators 2, 3, 4, 6, and 8) Develop understanding of fractions as numbers
Measurement • Describe and compare measurable attributes • Classify objects and count the number of objects in categories	Measurement and Data • Measure lengths indirectly and by iterating length units • Tell and write time • Represent and interpret data	Measurement and Data • Measure and estimate lengths in standard units • Relate addition and subtraction to length • Work with time and money • Represent and interpret data	Measurement • Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects • Represent and interpret data Geometric measurement: • Concepts of area and relate area to multiplication and to addition • Perimeter as an attribute of plane figures. Linear and area measures.
Geometry • Identify and describe shapes (circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and Spheres) • Analyze, compare, create, and compose shapes	Geometry • Reason with shapes and their attributes	Geometry • Reason with shapes and their attributes	Geometry • Reason with shapes and their attributes

[3] Adapted from: ed.sc.gov CCSS for Mathematics Resources

Problem Types and Complexity

Join	*(Result Unknown)* Connie had 5 marbles. Juan **gave** her 8 more. **How many** does Connie have altogether?	*(Change unknown)* Connie has 5 marbles. **How many** does she need to have 13 altogether?	*(Start Unknown)* Connie had some marbles. Juan gave her 5 more. Now she has 13. **How many** marbles did Connie have to start with?
Separate	*(Result Unknown)* Connie had 13 marbles. She **gave** 5 to Juan. How many marbles does Connie have left?	*(Change unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	*(Start Unknown)* Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-Part Whole	*(Whole Unknown)* Connie **has** 5 red marbles and 8 blue marbles. How many marbles does she have?	*(Part Unknown)* Connie has 13 marbles. 8 are blue. How many red marbles does Connie have?	
Compare	*(Difference Unknown)* Connie **has** 13 marbles. Juan has 5 marbles. **How many more** does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 5 marbles. Connie has 8 more. How many does she have?	*(Referent Unknown)* Connie has 13 marbles. She has 5 more than Juan. How many marbles does Juan have?

Multiplication and Division Problems

Type of CGI Problem	English Version	Spanish Version
Multiplication	Elena has 5 bags of cookies. There are 3 cookies in each bag. How many cookies does Elena have altogether?	Elena tiene 5 bolsas de galletas. En cada bolsa hay 3 galletas. ¿Cuántas galletas tiene en total?
Measurement Division	Elena has 15 cookies. She puts 3 cookies in each bag. How many bags can she fill?	Elena tiene 15 galletas. Ella pone 3 galletas en cada bolsa. ¿Cuántas bolsas puede llenar?
Partitive Division	Megan has 5 bags of cookies. Altogether she has 15 cookies. There are the same number of cookies in each bag. How many cookies are in each bag?	Elena tiene 5 bolsas de galletas. Ella tiene 15 galletas en total. En cada bolsa hay la misma cantidad de galletas. ¿Cuántas galletas hay en cada bolsa?

Adapted from Carpenter, et al. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

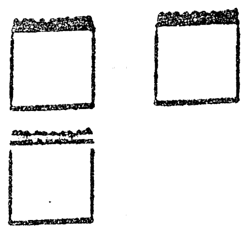
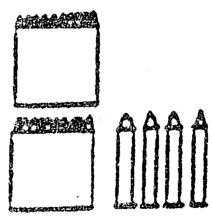
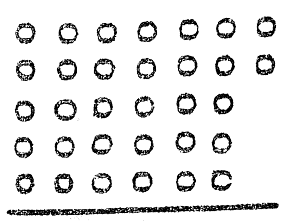
Problem Types and Examples	Direct Modeling Strategies	Advanced Strategies**	Comments
Join Change Unknown **Omar wants to buy a toy car that costs $11. He only has $7. How many more dollars does Omar need to buy the car?**	Joining To A set of 7 objects is constructed. Objects are added to this set until there is a total of 11 objects. The answer is found by counting the number of objects added.	Counting On To A forward counting sequence starts from 7 and continues until 11 is reached. The answer is the number of counting words in the sequence.	This is an action problem. Strong consistency in children's observed strategies.
Join Start Unknown **Daniela had some candies. Then her friend gave her 5 more and now she has 13. How many candies did Daniela have to start?**	Trial and Error A set of objects is constructed. A set of 5 objects is added to the set, and the resulting set is counted. If the final count is 13, then the number of objects in the initial set is the answer. If not, a different initial set is tried.	Trial and Error	This is an action problem, but it is hard to directly model because children do not know the starting value. Children were observed to use trial and error.
Compare **Gerardo has 12 pencils. His cousin Omar has 9 pencils. How many more pencils does Gerardo have than Omar.**	Matching A set of 12 objects and a set of 9 objects are matched 1-to-1 until one set is used up. The answer is the number of unmatched objects remaining in the larger set.	(no common strategy)	This is a relationship problem. A direct modeling strategy of matching was consistently observed, but no consistency was seen in counting strategies.
Part-Part-Whole **Gina has 10 balloons. Six of the balloons are blue and the rest are red. How many balloons are red?**	(no common strategy)	Counting On To (as above) Counting Down A backward counting sequence is initiated at 10 and goes for six counts. The last number in the counting sequence is the answer.	This is a relationship problem. Children employed a variety of ways to directly model, counting strategies showed more consistency when used.
Multiplication **Yolanda has three bags of marbles. There are seven marbles in each bag. How many marbles does Yolanda have altogether?**	Grouping Make 3 groups with 7 objects in each group. Count all the objects to find the answer.	Skip Counting (in specific cases) If numbers in each group are easily skip counted, children will count by these and may keep track with their fingers, e.g. 5s, 10s.	This is an action problem. There is high consistency in how children directly model it, but counting strategies are observed when the size of the groups is convenient for children to skip count.
Partitive Division **David has 15 marbles. He wants to share them with 3 friends so that each friend gets the same amount. How many does he give to each friend?**	Partitive Divide 15 objects into 3 groups with the same number of objects in each group. Count the objects in one group to find the answer.	(no common strategy)	This is an action problem. Counting strategies are difficult because children do not know the size of the groups. They tend to use trial and error to figure out what to count by.
**** For multidigit problems children will invent strategies that decompose numbers into tens and ones, increment in steps to reach a five or a ten, and/or compensate by solving with an easier number and then adjust their answer when they have reached a solution.**			

Adapted from Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Figure 4. Rate, Price, and Scaler Multiple Problems

Problem	Multiplication	Measurement Division	Partitive Division
Grouping/Partitioning	Gene has 4 tomato plants. There are 6 tomatoes on each plant. How many tomatoes are there altogether?	Gene has some tomato plants. There are 6 tomatoes on each plant. Altogether there are 24 tomatoes Howm many tomato plants does Gene have?	Gene has 4 tomato plants. There are the same number of tomatoes on each plant. Altogether there are 20 tomatoes. How many tomatoes are there on each tomato plant?
Rate	Ellen walks 3 miles an hour. How many miles does she walk in 5 hours?	Ellen walks 3 miles an hour. How many hours will it take her to walk 15 miles?	Ellen walked 15 miles. It took her 5 hours. If she walked the same speed the whole way, how far did she walk in one hour?
Price	Pies cost $4 each. How much do 7 pies cost?	Pies cost $4 each. How many pies can you buy for $28?	Jan bought 7 pies. He spent a total of $28. If each pie costs the same amount, houw much does one pie cost?
Scaler Multiple	The giraffe in the zoo is 3 times as tall as the kangaroo. The kangaroo is 6 feet tall. How tall is the giraffe?	The giraffe is 18 feet tall. The kangaroo is 6 feet tall. The giraffe is how many times taller than the kangaroo?	The giraffe is 18 feet tall. She is 3 times as tall as the kangaroo. How tall is the kangaroo?

Figure 4.4
Base Ten - Grouping/Partitioning Relationships

Grouping (No Extra)	Write the number. 	Carla has 3 boxes of crayons. There are 10 crayons in each box. How many crayons does Carla have altogether?	Carla had 3 boxes of crayons. There are 7 crayons in each box. How many crayons does Carla have altogether?
Grouping (Extra)		Mel has 2 boxes of crayons. There are 10 crayons in each box. He also has 4 extra crayons. How many crayons does he have in all?	Mel has 3 boxes of crayons. There are 7 crayons in each box. He also has 4 extra crayons. How many crayons does he have in all?
Partitioning (Measurement Division)	Circle groups of 10 and write the number.  How many tens are there in 32?	Josh has 32 ping pong balls. He wants to put them in boxes that hold 10 ping pong balls. How many boxes can he fill? How many balls are left over?	Josh has 32 ping pong balls. He wants to put them in boxes that hold 7 pong pong balls. How many boxes can he fill? How many balls are left over?

Types of Word Problems

Kinder

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A+B=\square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A+\square=C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square+B=C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C-B=\square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C-\square=A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square-B=A$
	Total Unknown	**Both Addends Unknown**[1]	**Addend Unknown**[2]
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A+B=\square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C=\square+\square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A+\square=C$ $C-A=\square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A+\square=C$ $C-A=\square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A+B=\square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C-B=\square$ $\square+B=C$

Mastered in 2nd

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

TASK LENS

Describe specific examples and provide evidence to support your responses.

1. *What makes this a good and/or problematic task for ELLs?* (e.g., multiple entry points, representations, solutions strategies; cognitive demand; home language – L1, math)

2. *What resources or knowledge does this task activate and/or connect to?* (e.g., mathematical, cultural, community, family, linguistic, students' interests, peers)

3. *How does the task structure create opportunities for ELLs to communicate mathematical thinking?*

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TASK LENS

Describe specific examples and provide evidence to support your responses.

Task Lens considerations for working with ELLs:

1. What makes this a good and/or problematic task for ELLs? Consider the following characteristics of mathematical tasks:

- **Multiple entry points:** Can students with varied language and math backgrounds begin work on the task, find ways to demonstrate their expertise, and contribute?
- **Multiple representations:** Does the task encourage use of representations such as physical objects (manipulatives), models, numerical/tabular approaches, graphical, geometric, symbols, etc.? Are there opportunities to make connections between various representations?
- **Multiple solution strategies:** Does the task encourage a variety of solution methods? Are students asked to solve a task in more than one way (e.g., verify their work by using a different approach)?
- **Cognitive demand:** Does the task require mathematical reasoning, problem solving, justification/explanations of thinking, etc. (high cognitive demand)? Or does the task focus more on practicing a procedure, drilling facts, etc. (low cognitive demand)?
- **Language (academic, everyday/conversational, home/L1):** Does the task support students' development of a mathematics language? Are students able to think and work in their first language? Are they able to make connections/distinctions between the mathematics language in the lesson and their everyday use of language (e.g., sphere and ball) and/or their home language (e.g., incorporating cognates)?

2. ***What resources or knowledge does this task activate and/or connect to?***

What forms of resources and knowledge are used/activated by the task? (e.g. mathematical, cultural, community, family, linguistic, students' interests, peers). Consider how these connections or lack of connections influence learning and what opportunities might exist to better connect the task to students.

3. ***How does the task structure create opportunities for ELLs to communicate mathematical thinking?***

Are students expected to show how they thought about the problem/solution? Does the nature of the task help the teacher and other students to understand and assess students' thinking as they work and/or after they have a solution? Describe examples when students' thinking is visible in the lesson.

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TEACHING LENS

Describe specific examples and provide evidence to support your responses.

1. ***What resources and knowledge does the teacher use/draw upon to support ELLs' mathematical understandings?*** (e.g., mathematical, cultural, community, family, linguistic, students' interests, peers)**.**

2. ***How does the teacher support opportunities for ELLs to communicate their mathematical understanding and participate in mathematical discourse?*** (e.g., eliciting ideas, publically position L1, affirm everyday language/experiences in explanations, recognize gestures as tools to communicate thinking, revoice student math ideas, directly model mathematical language)

3. ***How does the teacher respond to ELLs' mathematical ideas (correct answers, mistakes, confusions, partial understandings)?*** (e.g., follow-up clarifying questions; using students' home language and math language; using gestures or other representations; making mathematical connections among students' strategies)

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TEACHING LENS

Describe specific examples and provide evidence to support your responses.

Teaching Lens considerations for working with ELLs:

1. ***What resources and knowledge does the teacher use/draw upon to support ELLs' mathematical understandings?*** (e.g., mathematical, cultural, community, family, linguistic, students' interests, peers) What does the teacher do to help students make connections within mathematics (e.g., between various mathematical concepts, processes, representations, strategies, etc.)? What does the teacher do to help students connect mathematics with relevant/authentic issues or situations in their lives? How does the teacher draw upon family knowledge or experiences in the lesson? What does the teacher do to connect home language (L1)? What does the teacher do to encourage students to use their own prior knowledge and/or peers as resources for learning? Consider how these forms of resources/knowledge influence learning.

2. ***How does the teacher support opportunities for ELLs to communicate their mathematical understanding and participate in mathematical discourse?*** (e.g., eliciting ideas, publically position L1 as a resource, affirm everyday language/experiences in explanations, recognize gestures as tools to communicate thinking, revoice student math ideas, directly model mathematical language)**.** To what extent and how are various forms of communication supported, encouraged, and valued (e.g., verbal and non-verbal expressions, gestures, written work (symbols, pictures, diagrams), work with manipulative models/physical objects, etc.)? How does the teacher help to build (or diminish) students' sense of competence and self-confidence in mathematics?

3. ***How does the teacher respond to ELLs' mathematical ideas (correct answers, mistakes, confusions, partial understandings)?*** (e.g., follow-up clarifying questions; using students' home language and math language; using gestures or other representations; making mathematical connections among students' strategies)**.** What does the teacher do to help make students' mathematical thinking visible? How does the teacher handle student confusions/mistakes? How does the teacher handle correct answers? In what ways do repeating and/or restating an ELL's mathematics idea help clarify or extend a math idea?

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TEACHING LENS

Describe specific examples and provide evidence to support your responses.

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LEARNING LENS

Describe specific examples and provide evidence to support your responses.

1. ***What resources or knowledge do students draw upon to understand and solve the mathematics task?*** (e.g., mathematical, cultural, community, family, linguistic, students' interests, peers)

2. ***What specific mathematical understandings and/or confusions are indicated in students' work, talk, and/or behavior?***

3. ***How do ELLs communicate their mathematical understandings? How do ELLs make sense of others' mathematical ideas?*** (e.g., justifications, explanations, questions/responses, written work, representations, models, gestures)

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LEARNING LENS

Describe specific examples and provide evidence to support your responses.

Learning Lens considerations for working with ELLs:

1. What resources or knowledge do students draw upon to understand and solve the mathematics task?
(e.g., mathematical, cultural, community, family, linguistic, students' interests, peers)

What connections are students making between mathematics and their experiences, prior knowledge, and/or knowledge inside and outside of school? Consider how these forms of resources/knowledge influence learning.

2. What specific mathematical understandings and/or confusions are indicated in students' work, talk, and/or behavior?

Describe concepts, processes, skills, and/or problem solving strategies that students appear to know/understand as they engage in the lesson. Describe areas of confusion and/or misunderstanding. What did the students do, say, or show that indicated these understandings or confusions?

3. How do ELLs communicate what their mathematical understandings? How do ELLs make sense of other's mathematical ideas? (e.g., justifications, explanations, questions/responses, written work, representations, models, gestures)

How do students share their thinking? Consider evidence of understanding and/or learning in the form of: verbal and non-verbal expressions, gestures, written work (symbols, pictures, diagrams), work with physical objects (manipulatives, models), behavior, etc.

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POWER & PARTICIPATION LENS

Describe specific examples and provide evidence to support your responses.

1. ***Who participates in this mathematics lesson? Does the classroom culture value and encourage most students, including ELLs, to speak?*** (e.g., Do most students speak? Only a few students? Only the teacher?)

2. ***Who holds authority for knowing mathematics? The teacher? The students? Do some students' mathematical ideas hold more status than others?***

3. ***What evidence indicates that ELLs' mathematics contributions are/are not respected and valued?*** (e.g., Do ELLs share ideas publically, only with a teacher, only in small groups with peers? Do ELLs have opportunities to use home language (L1), mathematics language, everyday language, and/or code switching?)

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POWER & PARTICIPATION LENS

Describe specific examples and provide evidence to support your responses.

Power & Participation Lens considerations for working with ELLs:

1. Who participates in this mathematics lesson? Does the classroom culture value and encourage most students, including ELLs, to speak? (e.g., Do most students speak? Only a few students? Only the teacher?)

In lessons characterized by high levels of mathematical discourse, there is considerable teacher-student and student-student interaction about mathematical ideas; the interaction is reciprocal, and it promotes shared understandings. Observe these interactions and record how the teacher involves/calls on and whether students of various subgroups, including ELLs are represented.

2. Who holds authority for knowing mathematics? The teacher? The students? Do some students' mathematical ideas hold more status than others?

Consider how issues of authority and status might influence ELLs' confidence and sense of mathematical competence. Does the teacher have the final word on correct answers/solutions? Are students encouraged to convince themselves or others? Are some student's contributions given higher/lower status than others? Are some students assumed to "know" or "be correct" or "be better" at math? Are some students sought out or avoided as partners or group members? Who assigns high/low status (teacher, other students) and on what basis (academic performance, social standing, athletic performance)?

3. *What evidence indicates that ELLs' mathematics contributions are/are not respected and valued?*

Do ELLs share ideas publically, only with a teacher, only in small groups with peers? Do ELLs have opportunities to use home language (L1), mathematics language, everyday language, and/or code switching? Is there evidence that members are aware of and value different mathematics and personal experiences/background to support learning?

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Planning a Mathematics Lesson for ELLs

What are the big mathematical ideas of the lesson? What will students say or do that shows they understand these ideas? How can these ideas be connected to prior mathematical knowledge?

What do ELLs need to understand about the context to have access to the problem? What might work to verify that no misleading assumptions exist? What might be connections to students' lives, culture, and/or language? How could this be determined?

What specific language may need attention so the ELLs can understand the problem and develop mathematical understanding? What additional terminology is appropriate? Are there any gestures that can be used? What terms can be written on the chart that displays important terminology? What opportunities will there be for students to practice using this terminology as they participate in the Mathematics Discourse Community?

At what point will the ELLs work alone? In pairs? In groups? How will the students be grouped? What is the purpose of that grouping? How can you ensure that students actively participate, listen to each other, be respectful and inclusive of each other, and use each other as resources? How will students be positioned as competent problem solvers?

Are there explicit language and content objectives for specific ELLs? What serves as a foundation for these objectives? How do these objectives align with the trajectory planned for the specific students? At what point might it be appropriate for the students to rehearse responses or presentations?

Celedón-Pattichis, S., & Ramirez, N. G. (2012). *Beyond good teaching: Advancing mathematics education for ELLs*. Reston, VA: National Council of Teachers of Mathematics.
(Taken from Chapter 13, More4u materials)

What are different strategies, tools and/or representations students might use to complete this task?

What strategies will be used to assure that the ELLs have processing time? At what point(s) in the lesson will this occur?

What opportunities are there for the students to communicate orally? In written format?

Before working the problem

While working the problem

During discussion of solutions

During closure

Celedón-Pattichis, S., & Ramirez, N. G. (2012). *Beyond good teaching: Advancing mathematics education for ELLs*. Reston, VA: National Council of Teachers of Mathematics.
(Taken from Chapter 13, More4u materials)